

$$f = \{(-\infty, m), (-\infty, m-1), (0, m^r - 1)\}$$

۱)  $m-1 < m \Rightarrow -1 < 0$ : همواره برقرار است.

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$$2) m^r - 1 < m - 1 \Rightarrow m^r < m \Rightarrow m^r - m < 0 \Rightarrow m(m^{r-1} - 1) < 0$$

$$\begin{array}{c|ccccc} m & -\infty & -1 & 0 & 1 & +\infty \\ \hline m(m^{r-1} - 1) & - & + & 0 & - & + \end{array} \Rightarrow m < -1 \text{ یا } 0 < m < 1$$

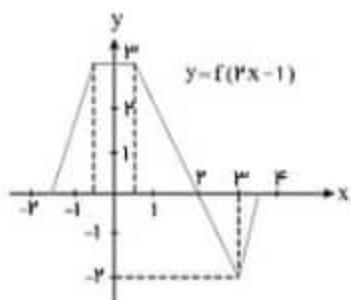
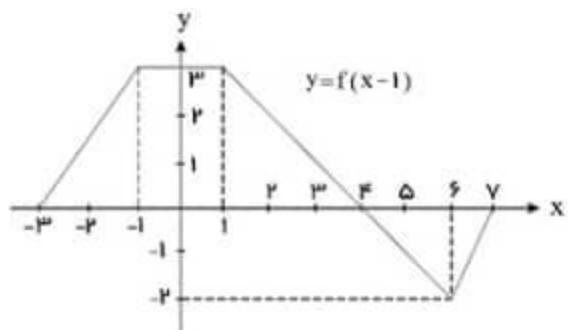
$$x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_f = (-\infty, 1] , D_g = R - \left\{ \frac{1}{r} \right\}$$

$$D_{fog} = \{x \in D_g ; g(x) \in D_f\} = \{x \in R - \left\{ \frac{1}{r} \right\}, \frac{x-1}{rx-1} \in (-\infty, 1]\}$$

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$$\frac{1}{rx-1} \leq 1 \Rightarrow \frac{1}{rx-1} \leq \frac{1}{r} \Rightarrow \frac{1}{rx-1} - \frac{1}{r} \leq 0 \Rightarrow \frac{r-rx+1}{r(rx-1)} \leq 0 \Rightarrow \frac{-rx+r+1}{r(rx-1)} \leq 0$$

$$\begin{array}{c|ccccc} x & - & \frac{1}{r} & 1 & + & - \\ \hline & - & + & 0 & - & \end{array} \Rightarrow D_{fog} = (-\infty, \frac{1}{r}) \cup [1, +\infty)$$



$$1 = r f(1) + 1 \Rightarrow f(1) = 0$$

$$rx + 1 = 1 \Rightarrow x = 0 , y = 1 - 1 = 0 \Rightarrow A'(0, 0)$$

$$f(x) = x^r + rx - 1 = (x+1)^r - 1$$

$$f(x_1) = f(x_r) \Rightarrow (x_1 + 1)^r - 1 = (x_r + 1)^r - 1 \Rightarrow |x_1 + 1| = |x_r + 1|$$

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$$\Rightarrow -(x_1 + 1) = -(x_r + 1) \Rightarrow x_1 = x_r$$

$$y = (x+1)^r - 1 \Rightarrow y + 1 = (x+1)^r \Rightarrow \sqrt{y+1} = |x+1| \Rightarrow$$

$$x+1 = -\sqrt{y+1} \Rightarrow x = -1 - \sqrt{y+1} \Rightarrow f^{-1}(x) = -1 - \sqrt{x+1}$$

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$$\frac{T}{\gamma} = \pi \Rightarrow T = \pi \Rightarrow \frac{\pi}{|b|} = \pi \Rightarrow |b| = \frac{1}{\pi} \Rightarrow b = \pm \frac{1}{\pi}$$

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$$\begin{cases} y_{\max} = |a| + c = \pi \\ y_{\min} = -|a| + c = -\pi \end{cases} \Rightarrow \pi c = -\pi \Rightarrow c = -1 \Rightarrow |a| = \pi \xrightarrow{a > 0} a = \pi$$

$$\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow \sin^2 22.5^\circ = \frac{1 - \cos 45^\circ}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4} \Rightarrow \sin 22.5^\circ = \frac{\sqrt{2} - \sqrt{2}}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \Rightarrow \cos^2 22.5^\circ = \frac{1 + \cos 45^\circ}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4} \Rightarrow \cos 22.5^\circ = \frac{\sqrt{2} + \sqrt{2}}{2}$$

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$$\sin x = 1 + \cos x \Rightarrow \tan \frac{x}{2} \cos \frac{x}{2} = \tan \frac{x}{2} \Rightarrow \begin{cases} \cos \frac{x}{2} = 1 \Rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \Rightarrow x = 2k\pi + \pi \\ \sin \frac{x}{2} = \cos \frac{x}{2} \Rightarrow \tan \frac{x}{2} = 1 = \tan \frac{\pi}{4} \Rightarrow \end{cases}$$

$$\frac{x}{2} = k\pi + \frac{\pi}{2} \Rightarrow x = 2k\pi + \pi$$

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$$\begin{cases} f(1) = 1 \Rightarrow 1 + m + 1 - n = 1 \\ f(-1) = -1 \Rightarrow -1 + m - 1 - n = -1 \end{cases} \Rightarrow \begin{cases} m - n = 1 \\ m - n = 1 \end{cases} \Rightarrow m = 1 \Rightarrow n = 0$$

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$$\lim_{x \rightarrow \infty} \frac{-x^2 + x + 1}{x^2 - x + 1} = \lim_{x \rightarrow \infty} \frac{-x^2 + x + 1}{(x-1)^2} = \frac{-\infty}{\infty} = -\infty$$

$$\lim_{x \rightarrow \infty} \frac{\Delta x + |x - 1|}{x - \sqrt{x^2 - 1}} = \lim_{x \rightarrow \infty} \frac{\Delta x + |x|}{x - |x|} = \frac{\Delta x - x}{x + x} = \frac{\Delta x}{2x} = \frac{1}{2}$$

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$$-rx^2 + ax + b = -r(x+1)^2 = -r(x^2 + 2x + 1) = -rx^2 - 2rx - r \Rightarrow a = -2r, b = -r$$

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$$ra - 1 = 0 \Rightarrow a = \frac{1}{r}, a + rb = 0 \Rightarrow \frac{1}{r} + rb = 0 \Rightarrow b = -\frac{1}{r}, \frac{rc}{c-r} = 1 \Rightarrow rc = r - 1 \Rightarrow c = -1$$

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مقدار مشتق تابع	-۳	-۱	.	۰/۵	۱	۲
نقطه متناظر	F	C	E	A	B	D

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$$f(1) = 1 - 1 + 1 = 1 \Rightarrow A(1, 1)$$

$$f'(x) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{x^2 - x - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)(x+1)}{x-1} = 2$$

$$y - y_A = m(x - x_A) \Rightarrow y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$$